Name: ____________________________

Instructor: ________________________

Math 10550, Practice Exam III
April 8, 2026

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 19 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)
11.	(a)	(b)	(c)	(d)	(e)
12.	(a)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
13.	______
14.	______
15.	______
Total	______

Name: ___________________________

Instructor: ________________________

Multiple Choice

1.(7 pts.) How many inflection points does the curve $y = \dfrac{x^4}{12} - \dfrac{x^3}{3}$ have?

Solution Notice that

$$y' = \frac{4}{12}x^3 - \frac{3}{3}x^2 = \frac{x^2}{3}(x-3)$$

and

$$y'' = \frac{3}{3}x^2 - 2x = x(x-2).$$

Thus, $y'' = 0$ if and only if $x = 0$ or $x = 2$. Observe that $y'' > 0$ if $x \in (-\infty, 0) \cup (2, \infty)$ and $y'' < 0$ if $x \in (0, 2)$. Given that there are change of sign in $x = 0$ and $x = 2$, we have that both are inflection points. Hence, y has 2 inflection points.

(a) 2 (b) 0 (c) 3 (d) 4 (e) 1

Name: ______________________________

Instructor: __________________________

2.(7 pts.) A bug being chased by a kitten (both moving in a straight line) enters a kitchen with velocity 1 ft/sec, and accelerates at $\frac{2}{\sqrt{t}}$ft/sec^2. How fast is the bug moving 9 seconds later.

Solution: Since acceleration is given by $a(t) = 2t^{-1/2}$ and the derivative of velocity is acceleration, we know that after integrating acceleration, the velocity is given by $v(t) = 4t^{1/2} + C$ for some constant C. We are also given that the initial velocity is 1 so that $v(0) = C = 1$. Thus, $v(t) = 4t^{1/2} + 1$. Thus, $v(9) = 12 + 1 = 13$.

(a) 5 ft/sec (b) 7 ft/sec (c) 13 ft/sec

(d) 4 ft/sec (e) 37 ft/sec

Name: ______________________________

Instructor: __________________________

3.(7 pts.) Suppose f is differentiable and $-2 \leq f'(x) \leq 1$ for all x and $f(2) = 3$. What are the minimum and maximum possible values for $f(5)$?

SOLUTION:

Since f is differentiable we can apply the Mean Value Theorem on $[2, 5]$ to get the existence of an $x \in (2, 5)$ such that

$$f'(x) = \frac{f(5) - f(2)}{5 - 2}$$

And by assumption (on f') we have,

$$-2 \leq \frac{f(5) - 3}{3} \leq 1$$

$$\implies -3 \leq f(5) \leq 6$$

(a) $-3 \leq f(5) \leq 0$

(b) $-5 \leq f(5) \leq 4$

(c) $3 \leq f(5) \leq 6$

(d) $-10 \leq f(5) \leq 10$

(e) $-3 \leq f(5) \leq 6$

Name: ____________________________

Instructor: ________________________

4.(7 pts.) Use linear approximation of $f(x) = \sqrt[3]{x}$ at $a = -8$ to estimate $\sqrt[3]{-8.12}$.

SOLUTION:

Using the linear approximation formula, we see that

$$L(x) = f(a) + f'(a)(x - a)$$

The derivative of f is $f'(x) = \dfrac{1}{3x^{2/3}}$. We see that $f(-8) = -2$ and

$$\begin{aligned} f'(-8) &= \frac{1}{3}\frac{1}{(-2)^2} \\ &= \frac{1}{12} \end{aligned}$$

So

$$L(x) = -2 + \frac{1}{12}(x + 8)$$

Thus, the approximation is

$$\begin{aligned} L(-8.12) &= -2 + \frac{1}{12}(-8.12 + 8.12) \\ &= -2 + -.01 \\ &= -2.01 \end{aligned}$$

(a) -2.2 (b) -2.01 (c) -1.8 (d) -1.99 (e) -2.04

Name: ____________________________

Instructor: ________________________

5.(7 pts.) Find the linearization of the function $f(x) = \sin^2(x)$ at $a = \dfrac{\pi}{4}$.

SOLUTION:

Recall that the "Linearization" of a function at a certain point a is just its tangent line at the given point; and it is given by

$$L(x) = f'(a)(x-a) + f(a).$$

Since $f'(x) = 2(\sin(x))\cos(x)$ we get $f'(a) = f'(\frac{\pi}{4}) = 2(\sin(\frac{\pi}{4}))\cos(\frac{\pi}{4}) = 2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} =$ 1. Also $f(a) = f(\frac{\pi}{4}) = \left(\sin(\frac{\pi}{4})\right)^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$. Hence,

$$L(x) = x - \frac{\pi}{4} + \frac{1}{2}.$$

(a) $x + \dfrac{1}{\sqrt{2}} - \dfrac{\pi}{4}$ (b) $-\dfrac{10}{4}x - \dfrac{1}{4}$ (c) $x + \dfrac{1}{2} - \dfrac{\pi}{4}$

(d) $\dfrac{1}{2}x + \dfrac{3}{2}$ (e) $\dfrac{x}{2} + \dfrac{1}{2} - \dfrac{\pi}{4}$

Name: ____________________________

Instructor: __________________________

6.(7 pts.) Which of the following gives a complete list of the critical numbers/values of the function

$$f(x) = 3x^{2/3} \cdot (x+1)^3$$

SOLUTION:

To be a critical number, you must either be a number a in the domain of f such that $f(a) = 0$ OR a number a in the domain of f such that f' does not exist at a. We observe that f is defined for every real number. Its derivative is given by

$$\begin{aligned} f'(x) &= \frac{2(x+1)^3}{x^{1/3}} + 9x^{2/3}(x+1)^2 \\ &= \frac{2(x+1)^3 + 9x(x+1)^2}{x^{1/3}} \end{aligned}$$

We see that f' is not defined at $a = 0$ and 0 is in the domain of f. So 0 is a critical number. Now, to find the rest of the critical values, we set $f' = 0$. We see this reduces to the equation

$$2(x+1)^3 + 9x(x+1)^2 = 0$$

This gives

$$(x+1)^2(2(x+1) + 9x) = 0$$

So $x = -1$ is a solution and solving

$$2(x+1) + 9x = 11x + 2 = 0$$

tells us that $x = -2/11$ is another solution. Thus, the full set of critical numbers is $x = 0, -2/11, -1$.

(a) $x = \frac{2}{11},\ -1$

(b) $x = \frac{-2}{11},\ -1$

(c) $x = \frac{2}{11},\ 1$

(d) $x = 0,\ \frac{-2}{11},\ -1$

(e) $x = -1, 0$

Name: ___________________________

Instructor: _______________________

7.(7 pts.) Let $f(x) = 4x^2 - 4x + 4$. Find the absolute maximum and absolute minimum of f on the interval $[0, 2]$. (That is find the maximum and minimum value of $f(x)$ on the given interval).

SOLUTION:

First we compute the derivative, $f'(x) = 8x - 4$. Setting $f'(x) = 0 = 8x - 4$, we see that $x = 1/2$ is the only critical number (note that $f'(x)$ is defined for every real number). The absolute maximum and absolute minimum must occur at a critical number or at an endpoint of the interval. So we simply evaluate f at each of these points. We see that

$$f(0) = 4$$

$$f(2) = 4 \cdot 4 - 4 \cdot 2 + 4 = 12$$

and

$$f(1/2) = 4(1/4) - 4(1/2) + 4 = 3$$

From our computations, we see that f has max value 12 and min value 3.

(a) Max value = 12, Min value = 4

(b) Max value = 4, Min value = 3

(c) Max value = 6, Min value = 4

(d) Max value = 12, No Minimum value exists

(e) Max value = 12, Min value = 3

Name: ____________________________

Instructor: ________________________

8.(7 pts.) Let

$$f(x) = \frac{x^3}{3} - 2x^2 - 12x + 17.$$

On which of the intervals given below is the graph of f **both** decreasing and concave down (on the entire interval)?

(a) $(6, \infty)$ (b) $(-2, 2)$ (c) $(-\infty, 2)$ (d) $(2, 6)$ (e) $(-2, 6)$

SOLUTION:

We are looking for an interval on which both f' and f'' are negative. We compute,

$$f'(x) = x^2 - 4x - 12 = (x - 6)(x + 2)$$

Notice that for any $x \in (-\infty, -2) \cup (6, \infty)$ we have $f'(x) > 0$. Also for $x \in (-2, 6)$ we have $f'(x) < 0$. We also compute,

$$f''(x) = 2x - 4.$$

Which is an increasing line with x-intercept at $x = 2$ and hence $f''(x) < 0$ for $x \in (-\infty, 2)$ and $f''(x) > 0$ for $x \in (2, \infty)$. By intersecting the two intervals of interest, i.e. those intervals for which both are negative, we get the interval $(-2, 2)$.

Name: ___________________________

Instructor: ________________________

9.(7 pts.) Consider the function $f(x) = x^4 - 8x^3 + 5$. Which of the following statements is true?

SOLUTION:

We begin by first computing the derivative and second derivative. We compute

$$\begin{aligned} f'(x) &= 4x^3 - 24x^2 \\ &= 4x^2(x-6) \end{aligned}$$

and

$$\begin{aligned} f''(x) &= 12x^2 - 48x \\ &= 12x(x-4) \end{aligned}$$

Now, we see that the zeros of $f''(x)$ are $0, 4$. When $x < 0$, we see that $f''(x)$ is positive by choosing any test point such as -1. We see that

$$\begin{aligned} f''(-1) &= 12(-1)(-1-4) \\ &= 60 \\ &> 0 \end{aligned}$$

When $0 < x < 4$, we see that $f''(x)$ is negative by choosing a test point such as 1. We see that

$$\begin{aligned} f''(1) &= 12(-3) \\ &= -36 \\ &< 0 \end{aligned}$$

When $x > 4$, we see that $f''(x)$ is positive by choosing a test point such as 5. We see that

$$\begin{aligned} f''(5) &= 5(12 \cdot 5 - 48) \\ &= 5 \cdot 12 \\ &= 60 \\ &> 0 \end{aligned}$$

Our computations show that f'' changes signs at $x = 0$ and $x = 4$. This tells us that 0 and 4 are both points of inflection. Now, the zeros of the derivative (i.e, the critical numbers of f) are 0 and 6. We see that $f''(6) > 0$ and so the second derivative test tells us that at $x = 6$, there is a local minimum. By choosing test points such as -1 and 1, we see that $f'(1) = -20 < 0$ and $f'(-1) = -28 < 0$. Thus, at the point $x = 0$, there is neither a local minimum nor a local maximum.

Name: ______________________________

Instructor: __________________________

(a) f has a local maximum at $x = 0$, no local minimum, and a point of inflection at $x = 4$.

(b) f has a local minimum at $x = 6$, no local maximum, and points of inflection at $x = 0, 4$ and -4.

(c) f has a local minimum at $x = 6$, no local maximum, and points of inflection at $x = 0$ and 4.

(d) f has local minima at $x = 0$ and 6, no local maximum, and a point of inflection at $x = 4$.

(e) f has a local minimum at $x = 6$, a local maximum at $x = 0$, and points of inflection at $x = 0$ and 4.

Name: ______________________________

Instructor: __________________________

10.(7 pts.) $\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}} =$

- This is an indeterminate form of type 1^∞.
- We have

$$\lim_{x\to 0^+} (\cos x)^{1/x^2} = \lim_{x\to 0^+} Exp\left[\frac{\ln(\cos x)}{x^2}\right] = Exp\left[\lim_{x\to 0^+} \frac{\ln(\cos x)}{x^2}\right]$$

($Exp[x] = e^x$ here)

- $= \quad (byl'Hop) \quad Exp\left[\lim_{x\to 0^+} \frac{\frac{1}{\cos x}(-\sin x)}{2x}\right] = Exp\left[\lim_{x\to 0^+} \frac{-\tan x}{2x}\right]$
- $= \quad (byl'Hop) \quad Exp\left[\lim_{x\to 0^+} \frac{-\sec^2 x}{2}\right] = e^{-1/2}$

(a) ∞ (b) e (c) 1

(d) $e^{-\frac{1}{2}}$ (e) Does not exist

Name: ____________________________

Instructor: _________________________

11.(7 pts.) Calculate the following indefinite integral $\int \frac{1+x}{\sqrt{x}} dx$ $\Bigg($i.e. find the general antiderivative for the function $f(x) = \frac{1+x}{\sqrt{x}}\Bigg)$.

$$\int \frac{1+x}{\sqrt{x}} dx = \int x^{-1/2} + x^{1/2} dx = 2x^{1/2} + \frac{2x^{3/2}}{3} + C$$

(a) $\frac{\sqrt{x}}{2} + \frac{3x^{3/2}}{2} + C$

(b) $\sqrt{x} + x^{3/2} + C$

(c) $2\sqrt{x} + \frac{2x^{3/2}}{3} + C$

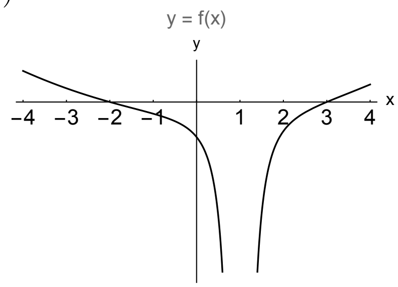
(d) $\frac{-1}{2x^{3/2}} - \frac{1}{\sqrt{x}} + C$

(e) $\frac{-1}{2x^{3/2}} + \frac{1}{\sqrt{x}} + C$

Name: ______________________________

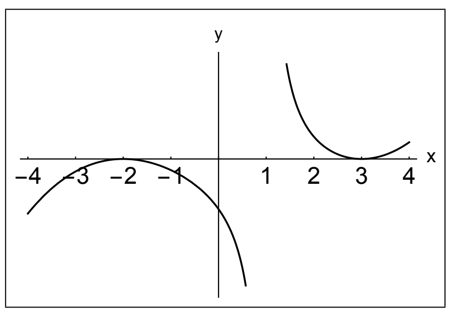
Instructor: __________________________

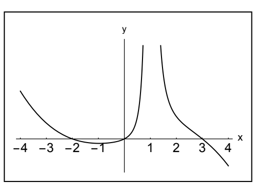
12.(7 pts.) The graph of $f'(x)$ is shown below:

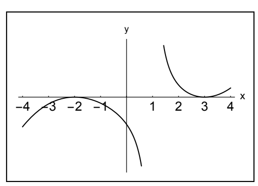


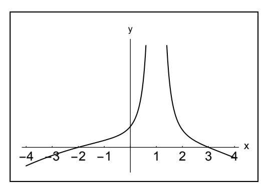
which of the following gives a feasible graph of $f(x)$?

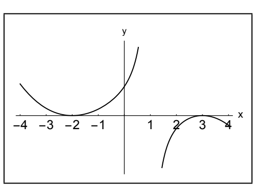
Solution We have $f'(x) > 0$ on the intervals $(-4, -2)$ and $(3, 4)$ and is < 0 on the intervals $(-2, 1)$ and $(1, 3)$. Therefore $f(x)$ is increasing in $[-4, -2]$ and $[3, 4]$; decreasing in $[-2, 1) \cup (1, 3]$. The answer is

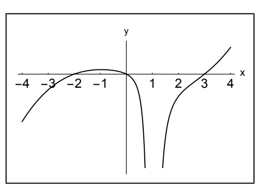


(a) 

(b) 

(c) 

(d) 

(e) 

Name: ____________________________

Instructor: ________________________

Partial Credit

You must show your work on the partial credit problems to receive credit!

13.(12 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

Solution Let x denote the total width and y denote the total height. So the width of the printed area is $x-2$ and the height of the printed area is $y-3$. Then the total area of the page can be expressed as

$$A_{total} = xy.$$

We are given that $A_{total} = 150$, so $y = 150/x$. We wish to maximize

$$A_{print} = (x-2)(y-3) = (x-2)\left(\frac{150}{x} - 3\right) = 156 - 3x - \frac{300}{x}.$$

Differentiating with respect to x and finding critical points gives

$$A'_{print}(x) = -3 + \frac{300}{x^2} = 0$$

so we must have $300 - 3x^2 = 0$, i.e. $x^2 = 100$. So $x = 10$ inches.

Using the first derivative test shows that 10 is indeed a maximum. For $x < 10$, $A'_{print} > 0$, and for $x > 10$, $A'_{print} < 0$.

$y = 150/x$, so we have $y = \dfrac{150}{10} = 15$. Therefore the page which maximizes the printed area has the dimensions 10 inches by 15 inches.

Name: ______________________________

Instructor: __________________________

14.(12 pts.) Consider the function

$$f(x) = 2x + \sin(x) + x^3 + 2$$

Show that $f(x)$ has *exactly one* root (i.e. exactly one solution to the equation $f(x) = 0$).

Solution: Notice that $f(0) > 0$ and $f(-\pi/2) < 0$, so by the Intermediate Value Theorem, f has at least one root between $-\pi/2$ and zero.

Assume that there are two distinct solutions, c_1 and c_2, to the equation $2x + \sin(x) + x^3 + 2 = 0$; that is $f(c_1) = f(c_2) = 0$ for distinct c_1 and c_2. Then, since f is differentiable everywhere (in particular, f is differentiable on (c_1, c_2) and continuous on $[c_1, c_2]$), by the mean value theorem there's some $x \in [c_1, c_2]$ such that

$$f'(x) = \frac{f(c_2) - f(c_1)}{c_2 - c_1} = 0.$$

The derivative of $f(x)$ is

$$f'(x) = 2 + \cos(x) + 3x^2.$$

$f'(x)$ is always greater than 0 since $\cos(x) > -1$ and $f'(x) > 2+(-1)+0 = 1$. Hence, $f'(x)$ has no zeroes, giving us a contradiction. Therefore so our initial (and only) assumption that we have two distinct roots c_1 and c_2 is false. Thus we can conclude that there is exactly one solution to the equation.

Name: ____________________

Instructor: ____________________

15. (4 pts.) Please circle "TRUE" if you think the statement is true, and circle "FALSE" if you think the statement is False.

(a)(1 pt. No Partial credit) By the Intermediate Value Theorem, the function $f(x) = \dfrac{1}{\sin(x)}$ must have an absolute maximum on the interval $-1 \leq x \leq 1$.

FALSE : $f(x)$ is not continuous at $x = 0$ and hence the I.V.T. does not apply.

TRUE FALSE

(b)(1 pt. No Partial credit) If c is a critical value for $f(x)$ with $f'(c) = 0$ and $f''(c) > 0$, then $f(x)$ must have a local minimum at $x = c$.

TRUE by the second derivative test.

TRUE FALSE

(c))(1 pt. No Partial credit) The general antiderivative of $f(x) = \ln(x)$ is $\dfrac{1}{x} + C$..

FALSE $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = \dfrac{-1}{x^2}$, so $\dfrac{1}{x}$ is not an antiderivative for $\ln(x)$.

TRUE FALSE

(d))(1 pt. No Partial credit) If $f''(x) > 0$ on the interval $-1 \leq x \leq 1$, then using linear approximation at $x = 0$ to estimate $f(0.1)$ gives an underestimate of the value of $f(0.1)$.

TRUE; because the tangents to the graph on the interval $(-1, 1)$ lie below the graph.

TRUE FALSE

Name: ______________________________

Instructor: __________________________

The following is the list of useful trigonometric formulas:

Note: $\sin^{-1} x$ and $\arcsin(x)$ are different names for the same function and $\tan^{-1} x$ and $\arctan(x)$ are different names for the same function.

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$a^x = e^{x\ln(a)}$$

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

Name: ______________________________

Instructor: __________________________

Rough Work

Name: ____________________________

Instructor: ANSWERS

Math 10550, Practice Exam III
April 8, 2026

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 19 pages of the test.
- Each multiple choice question is worth 7 points. Your score will be the sum of the best 10 scores on the multiple choice questions plus your score on questions 13-15.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(•)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(•)
4.	(a)	(•)	(c)	(d)	(e)
5.	(a)	(b)	(•)	(d)	(e)
6.	(a)	(b)	(c)	(•)	(e)
7.	(a)	(b)	(c)	(d)	(•)
8.	(a)	(•)	(c)	(d)	(e)
9.	(a)	(b)	(•)	(d)	(e)
10.	(a)	(b)	(c)	(•)	(e)
11.	(a)	(b)	(•)	(d)	(e)
12.	(a)	(•)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
13.	______
14.	______
15.	______
Total	______